Name: _____

Calculus I

Professor Piotr Hajłasz

First Exam

October 7, 2009, 1pm.

Problem	Possible points	Score
1	20	
2	30	
3	10	
4	20	
5	20	
Total	100	

Exercise 1. (20=2×10pts.) Find the following limits

(a) $\lim\limits_{x\to 0} \dfrac{x^2 \cos x}{\sin(x^2)}$

(b) $\lim\limits_{x\to\infty} \dfrac{x^2 - 4x + 7}{2x^2 + x \cos x}$

Exercise 2. (30=3×10pts.)

(a) Find the derivative of $f(x) = \cos(\sin^2(x^2))$

(b) Find the derivative of $h(t) = \dfrac{t + \cos^2 t}{2 - \sqrt{t}}$.

(c) Use the definition to compute the derivative of the function $f(x) = \dfrac{1}{3x - 1}$.

Exercise 3. (10 pts) Determine values of a and b that make the given function continuous.

$$f(x) = \begin{cases} \frac{3\sin x}{x} & \text{if } x > 0, \\ a & \text{if } x = 0 \\ b(x^2 + 1) & \text{if } x < 0 \end{cases}$$

Exercise 4.(20 pts) Find the equation of the tangent line to the curve $x^2y^2 - 2x = 4 - 4y$ at the point $(2, -2)$.

Exercise 5. (20pts) A car is traveling at 50 mph south at a point 0.5 mile north of an intersection. A police car is traveling at 40 mph due west at a point 0.25 mile east of the same intersection. At this instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register?